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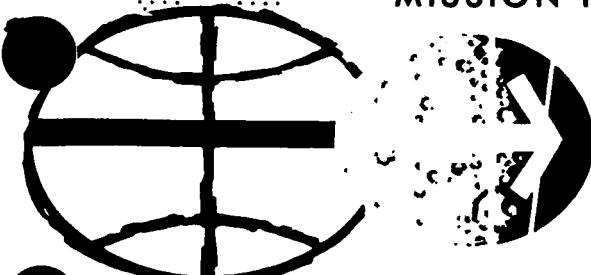
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BASIC CHARACTERISTICS OF PARKING
ORBITS ABOUT MARS FOR USE IN
PRELIMINARY MISSION PLANNING

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Orbital Mission Analysis Branch
MISSION PLANNING AND ANALYSIS DIVISION



MANNED SPACECRAFT CENTER
HOUSTON, TEXAS

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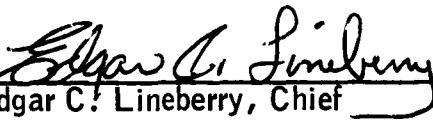
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BASIC CHARACTERISTICS OF PARKING ORBITS ABOUT
MARS FOR USE IN PRELIMINARY MISSION PLANNING

By Richard J. Carr and Alexander Woronow
Orbital Mission Analysis Branch

November 28, 1969

MISSION PLANNING AND ANALYSIS DIVISION
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BASIC CHARACTERISTICS OF PARKING ORBITS ABOUT MARS

FOR USE IN PRELIMINARY MISSION PLANNING

By Richard J. Carr and Alexander Woronow

1.0 SUMMARY AND INTRODUCTION

The purpose of this note is to present preliminary Mars data constants and parametric trajectory plots from which information (particularly rendezvous parameters) for early mission planning of Mars orbits can be obtained. Some Earth constants are also included for reference. A Keplerian method was used to generate most of the data; consequently, except for calculations of nodal and apsidal regression, small perturbations in the orbit have been ignored. Only data for circular orbits have been included, with the exception of data which indicate sensitivity of Hohmann transfers in elliptic orbits. Generally, orbit altitudes of between 100 n. mi. and 1000 n. mi. were used, although higher orbits may be flown for missions that involve the Mars moons, Phobos and Deimos, which orbit Mars at 5065 n. mi. and 12 682 n. mi., respectively.

Information pertinent to specific rendezvous missions is not included at this time; however, data for the basic coelliptic rendezvous technique are given. No data pertinent to Earth-to-Mars transfer trajectories are included.

2.0 SYMBOLS

J_2	dynamical form factor, or oblateness coefficient of Mars
a	semimajor axis, n. mi.
μ	gravitational constant
r	radius of satellite in its orbit, ft
v	circular velocity, fps

R_m	mean radius of Mars, ft
ϕ	angle between the radius vector to the Mars surface terminator and the radius vector to the satellite terminator
θ	half angle between radius vectors at entrance and exit to shadow
t_d	time spent in darkness, min
p	period of satellite about Mars, min
ΔH	height differential between concentric Martian orbits, n. mi.
Δh	height differential between apoapsis of an elliptic orbit and a circular orbit having the same periapsis altitude, n. mi.
ΔP	difference in orbital periods or catchup rate, deg/orb
ΔV_{pc}	in-orbit velocity increment required to change plane
δ	dihedral angle between orbital planes of two Martian orbits
e	eccentricity
k	conversion factor = 6076.115 ft/n. mi.
r_a	radius at apoapsis, n. mi.
r_p	radius at periapsis, n. mi.
Ω	ascending node of the orbital plane, deg
i	inclination of orbit to Mars equator, deg
g	argument of periapsis, deg

3.0 DISCUSSION

A brief list of Earth, Mars and Mars Moon constants is given in table I. The oblateness of Mars can be computed through observation of the equatorial and polar semidiameters, known as geometric flattening, or by observation of the action of the equatorial bulge on the motion of Mars' moons, known as dynamic flattening. The dynamic value of 0.00197

was used in this note rather than the geometric flattening value (0.0055). Because the latter value is more than twice the other value, it is difficult to determine which value is more valid at this time.

Circular orbits were considered with one exception. The orbital period is shown as a function of altitude in figure 1. Altitude is given in n. mi. above the mean radius of Mars in this figure and in all others. The formula used to compute the period is

$$P = 2\pi a \sqrt{\frac{a}{\mu}}$$

where $a = r$ for circular orbits.

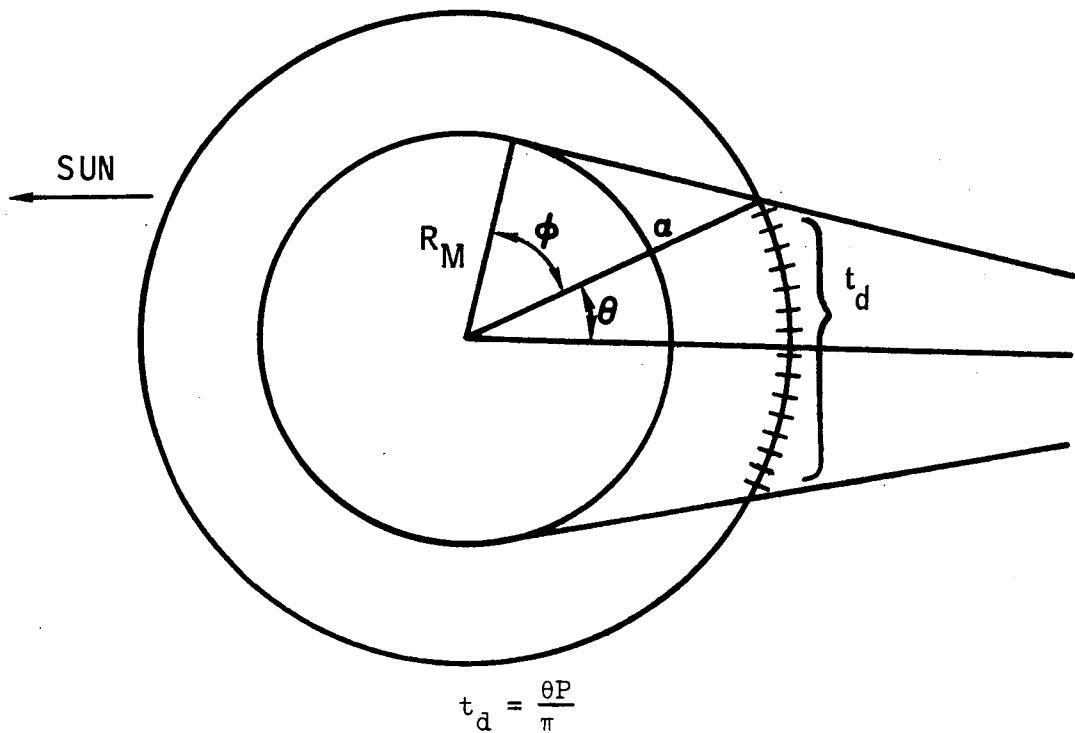
The circular velocity at various altitudes is shown in figure 2 for orbits up to 1000 n. mi. circular. The formula used to compute circular velocity is

$$v = \sqrt{\frac{\mu}{r}}$$

Arc length per degree of central angle is shown in figure 3 where altitude is again the controlling variable. Arc length is computed from $\frac{\pi r}{180}$ where r is given in n. mi. above the center of Mars.

The mean angular motion in deg/min for circular orbits is shown in figure 4. The angular motion is 360° divided by the period. Data used to determine the length of time per degree of central angle travelled for altitudes of up to 2000 n. mi. are presented in figure 5. To determine this length of time, divide the period of the orbit by 360.

The amount of time per orbit that the spacecraft will remain in darkness can be found from data in figure 6. To compute the time, it was assumed that the vehicle orbital plane coincides with the Mars orbital plane about the Sun which yields maximum darkness conditions per orbit. Orbits of up to 1000 n. mi. above the mean radius of Mars were considered. The time in darkness is derived as follows.



where P = orbital period

$$\cos \phi = \frac{R_m}{a}$$

Because of the great distance from the Sun to Mars

$$\theta \approx \frac{\pi}{2} - \phi$$

$$\theta \approx \frac{\pi}{2} - \cos^{-1} \left(\frac{R_m}{a} \right)$$

therefore,

$$t_d = \left[\frac{\pi}{2} - \cos^{-1} \left(\frac{R_m}{a} \right) \right] \frac{P}{\pi}$$

where t_d is time in darkness, min.

To find the percentage of the orbit about Mars that will be in darkness, divide the results from figure 6 by the period for the specific orbit. These values are shown as percentages in figure 7. Again, the vehicle's orbital plane is considered coincident with the Mars orbital plane.

The catchup rate is shown in figure 8 as a function of the difference in average altitudes (Δh) of the Mars orbits of the chaser and target vehicles. Separate curves are shown for several target altitudes. To compute the catchup rate, the following expression was used.

$$\Delta P = \frac{3}{4} \frac{\Delta h \sqrt{\mu}}{\pi a^{5/2}}$$

The change in velocity required to remove various wedge angles (dihedral angle between orbital planes) is shown in figure 9 as a function of the circular altitude of the vehicle. The following formula was used

$$\Delta V_{pc} = 2V \sin \frac{\delta}{2}$$

where δ is the dihedral angle between the two orbital planes.

The coefficient of the $\Delta h / \Delta V$ ratio for various circular orbits of up to 1000 n. mi. above Mars can be observed in figure 10. Note that the coefficient varies considerably with altitude such that an average value should be used with care. The derivation of this coefficient is explained in the following paragraphs.

Propulsion added inplane and tangential to the orbital path results in a change in orbital altitude (Δh) on the diametrically opposite side of the orbit. With a simple relationship, a change in altitude can be determined as a function of the impulse magnitude.

$$v^2 = \mu \left(\frac{2}{r} - \frac{1}{a} \right)$$

where for circular orbits $a = r$. Because $da = \frac{dh}{2}$

$$2V dV = \frac{da}{a^2} = \mu \frac{da}{a^2} = \mu \frac{dh}{2a^2}$$

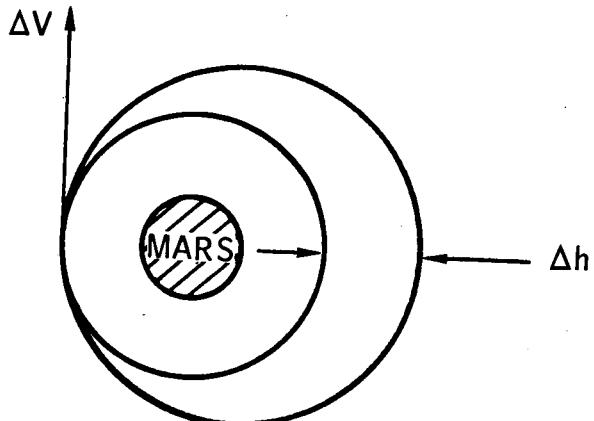
For $e = 0, a = r$

$$v^2 a = \mu$$

$$dh = \frac{4r^2}{V} v dV$$

$$dh \approx \frac{4a}{V} dv$$

As an example, the following calculation involves a 100-n. mi. circular orbit.



$$V = 11\ 393 \text{ fps}$$

$$a = 1925.2 \text{ n. mi.}$$

$$\Delta h = \frac{4a}{V} \Delta V$$

$$\Delta h = 0.68 \Delta V$$

Δh = altitude change at one end of the orbit, n. mi.

ΔV = magnitude of impulse, fps

The change in coefficient of the $\Delta h/\Delta V$ ratio is shown in figure 11 for Hohmann transfers from elliptical Mars orbits. To compute the elliptic injection coefficient in n. mi./fps, the following expression was used.

$$\frac{dr_a}{dV_p} = \frac{4a}{K} \sqrt{\frac{ar_a}{\mu r_p}}$$

To compute the circulation in n. mi./fps, the following expression was used.

$$\frac{dr_p}{dV_a} = \frac{4r_a}{K \sqrt{\frac{\mu}{r_a}}}$$

For both expressions, $k = 6076.115 \text{ ft/n. mi.}$, $a = \frac{r_a + r_p}{2}$,
 r_a = radius at apoapsis, r_p = radius at periapsis.

The orbital regression rate is shown in figure 12 for various inclinations. The formula used to compute the family of curves is as follows.

$$\frac{d\Omega}{dt} = - \frac{3}{2} J_2 \frac{R_m^2 \sqrt{\mu}}{(1 - e^2)^2 a^{7/2}} \cos i$$

where

Ω = ascending node, deg

i = inclination of orbit to Mars equator, deg

R_m = mean radius of Mars, ft

$a = r$ = circular orbit radius, ft

Note that the J_2 used (0.00197) is the dynamic value and that this value differs by a factor of 2.5 from the J_2 determined by observation of actual Mars shape (0.0055); thus, these regression rates may be in error by approximately this factor.

The inplane apsidal advance is shown in figure 13 at several inclinations for circular Mars orbits up to 1000 n. mi. above the mean Mars surface. The following equation was used to compute the family of curves.

$$\frac{dg}{dt} = \frac{3}{4} J_2 \frac{R_m^2 \sqrt{\mu}}{a^{7/2}(1 - e^2)^2} (5 \cos^2 i - 1)$$

where $J_2 = 0.00197$

i = inclination

R_m = radius of Mars, ft

$a = r$

4.0 CONCLUDING REMARKS

These data have been presented for use in preliminary planning of Mars orbital missions. The Mars constants may change with future studies. Specific rendezvous techniques were not included in this note.

TABLE I.- PHYSICAL CONSTANTS AND CHARACTERISTICS

(a) Earth data

Equatorial radius: 3963.2080 mi. = 3443.9335 n. mi.

Mean radius: 3959.7000 mi. = 3440.8237 n. mi.

Gravitational acceleration (at surface, 45° latitude): 32.1740 fps

$\mu_{\text{Earth}}: 6.2749 \times 10^4 \text{ n. mi.}^3/\text{sec}^2 = 1.407654 \times 10^{16} (\text{Int ft})^3/\text{sec}^2$

Mass of Earth: $13.1726 \times 10^{24} \text{ lb} = 5.975 \times 10^{24} \text{ kg}$

Earth rotational angular velocity: $7.292115 \times 10^{-5} \text{ rad/sec}$

Eccentricity: 0.0167

Circular Velocity at equator: $2.5950 \times 10^4 \text{ fps}$

Escape velocity at equator: $3.6645 \times 10^4 \text{ fps} = 6.031 \text{ n. mi./sec}$

(b) Mars data

Radius: 2100.39 mi. = 1825.1885 n. mi. = 0.52997 e.r.

Gravitational acceleration (at surface): $0.38 g_e = 12.236 \text{ ft/sec}^2$

Mass of Mars: 0.108 (Earth) = $1.4226 \times 10^{24} \text{ lb}$

Escape velocity (at surface): $2.7244 \text{ n. mi./sec} = 1.67 \times 10^4 \text{ fps}$

Eccentricity of orbit: 0.093368

Period of rotation: 24 hr, 37 min, 23 sec = 24.62304 hr

Inclination of equator to ecliptic: 27.05°

Period of revolution: 1.8822 Earth years = 1 year 322 days

$\mu_{\text{Mars}}: 4.3 \times 10^4 \text{ km}^3/\text{sec}^2 = 0.667 \times 10^4 \text{ n. mi.}^3/\text{sec}^2$

TABLE I.- PHYSICAL CONSTANTS AND CHARACTERISTICS - Concluded

Semimajor axis of Mars orbit about Sun: 1.523691 AU

Sphere of influence of Mars: 1.2985×10^5 km = 7.0113×10^4 n. mi.

J_2 : 0.00197 (dynamic flattening) or 0.0055 (geometric flattening)

(c) Mars satellite data

Mars I (Phobos)

Semimajor axis of orbit about Mars: 6.27×10^{-5} AU = 5064.68 n. mi.

Period: 0.319 Earth days

Radius: 6 n. mi.

Inclination to Mars equatorial plane: 1°

Mars II (Deimos)

Semimajor axis of orbit about Mars: 1.570×10^{-4} AU = 12681.89 n. mi.

Period: 1.262 Earth days

Radius: 3 n. mi.

Inclination to Mars equatorial plane: 1.75°

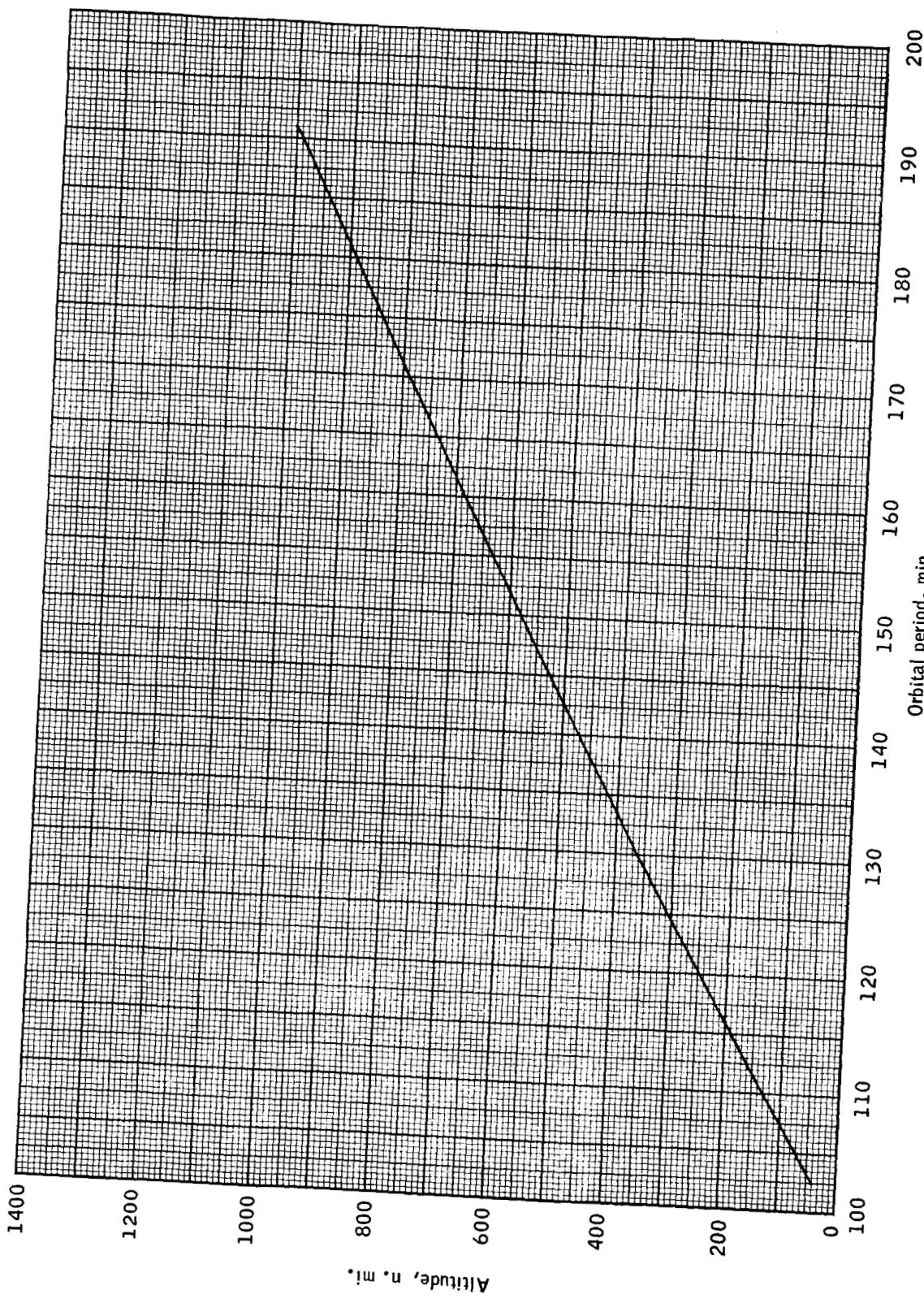


Figure 1.- Circular altitude versus orbital period for circular orbits around Mars.

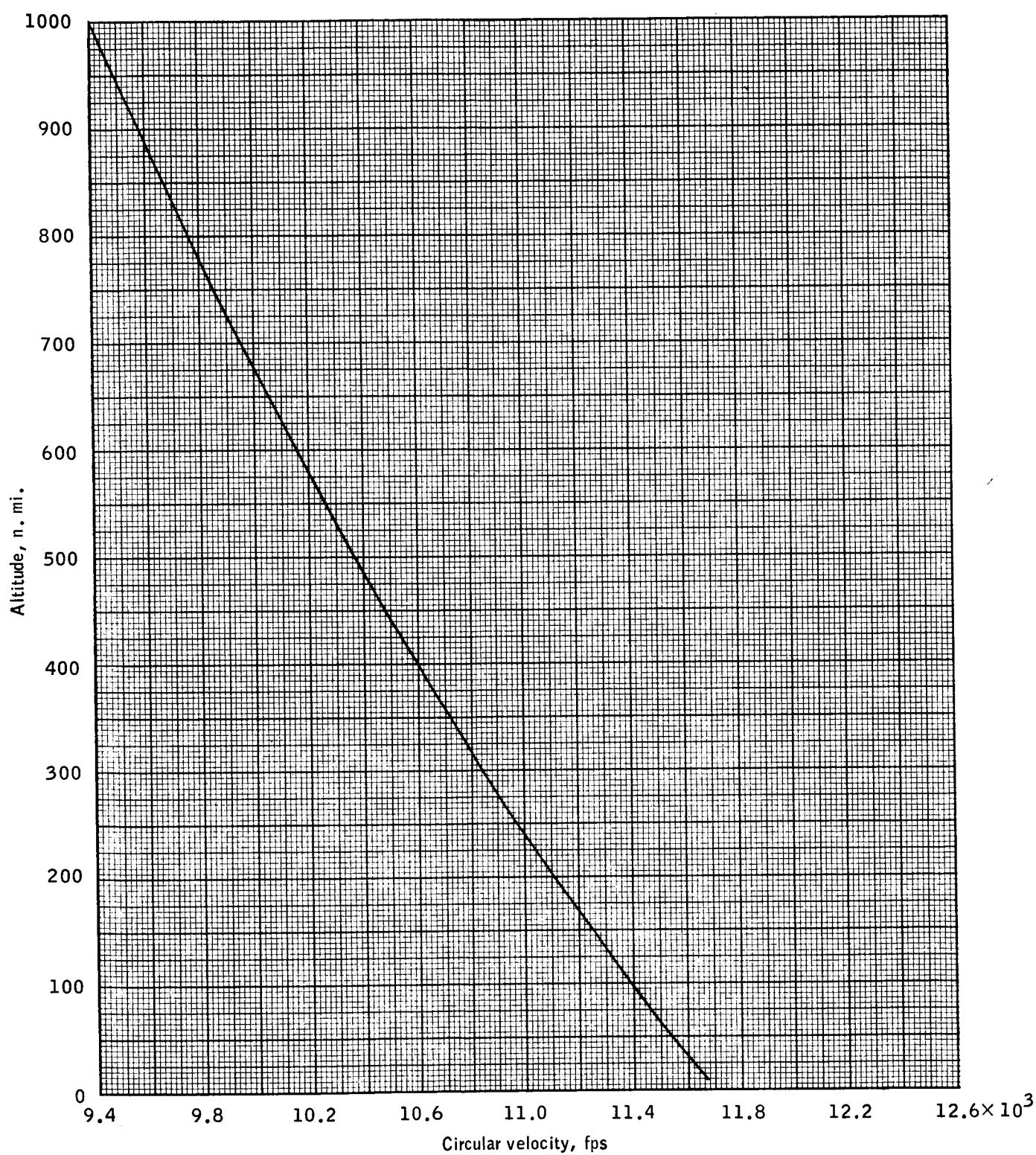


Figure 2.- Altitude versus velocity for circular orbits around Mars.

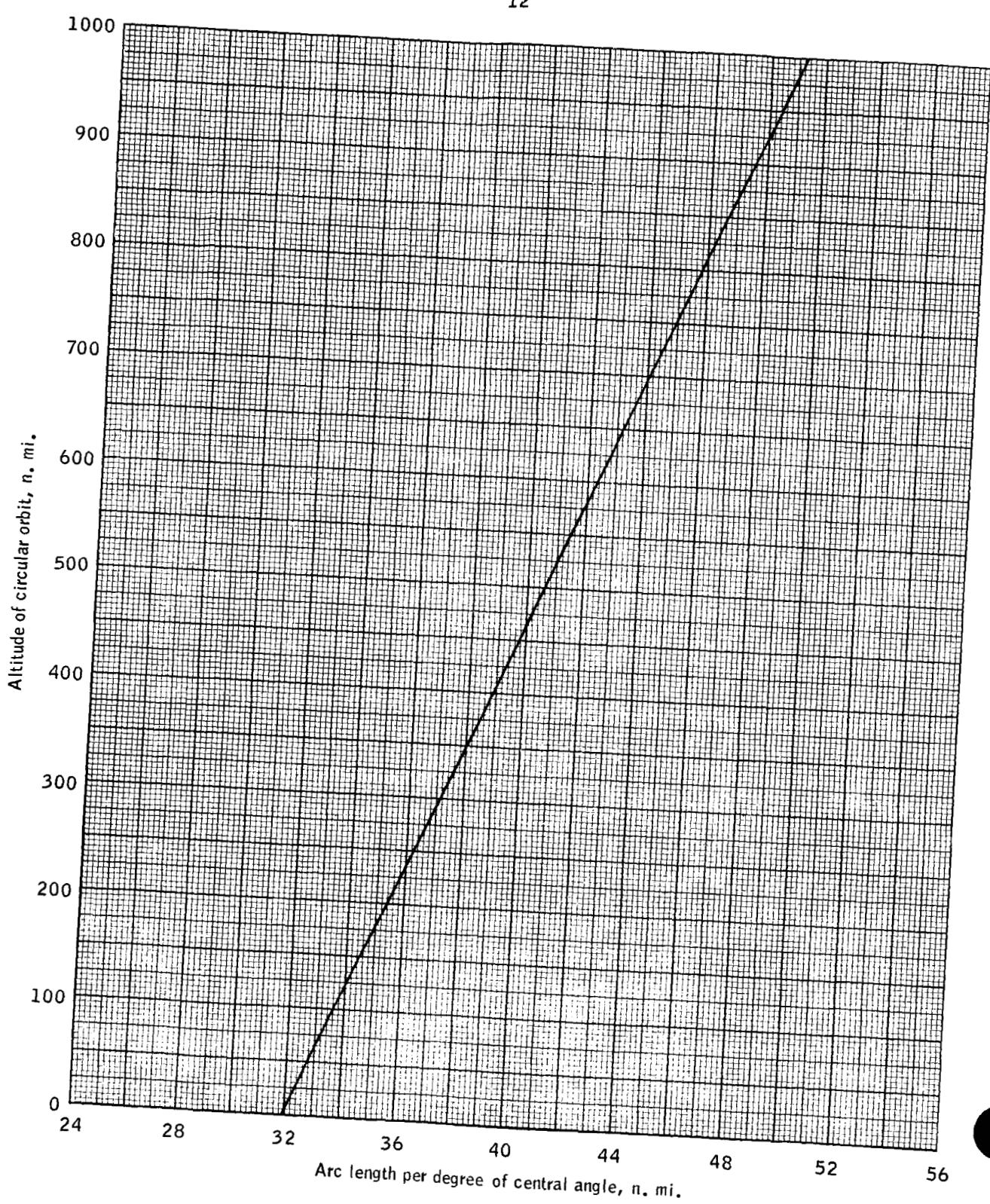


Figure 3.- Altitude of circular Mars orbit versus arc length per degree of central angle.

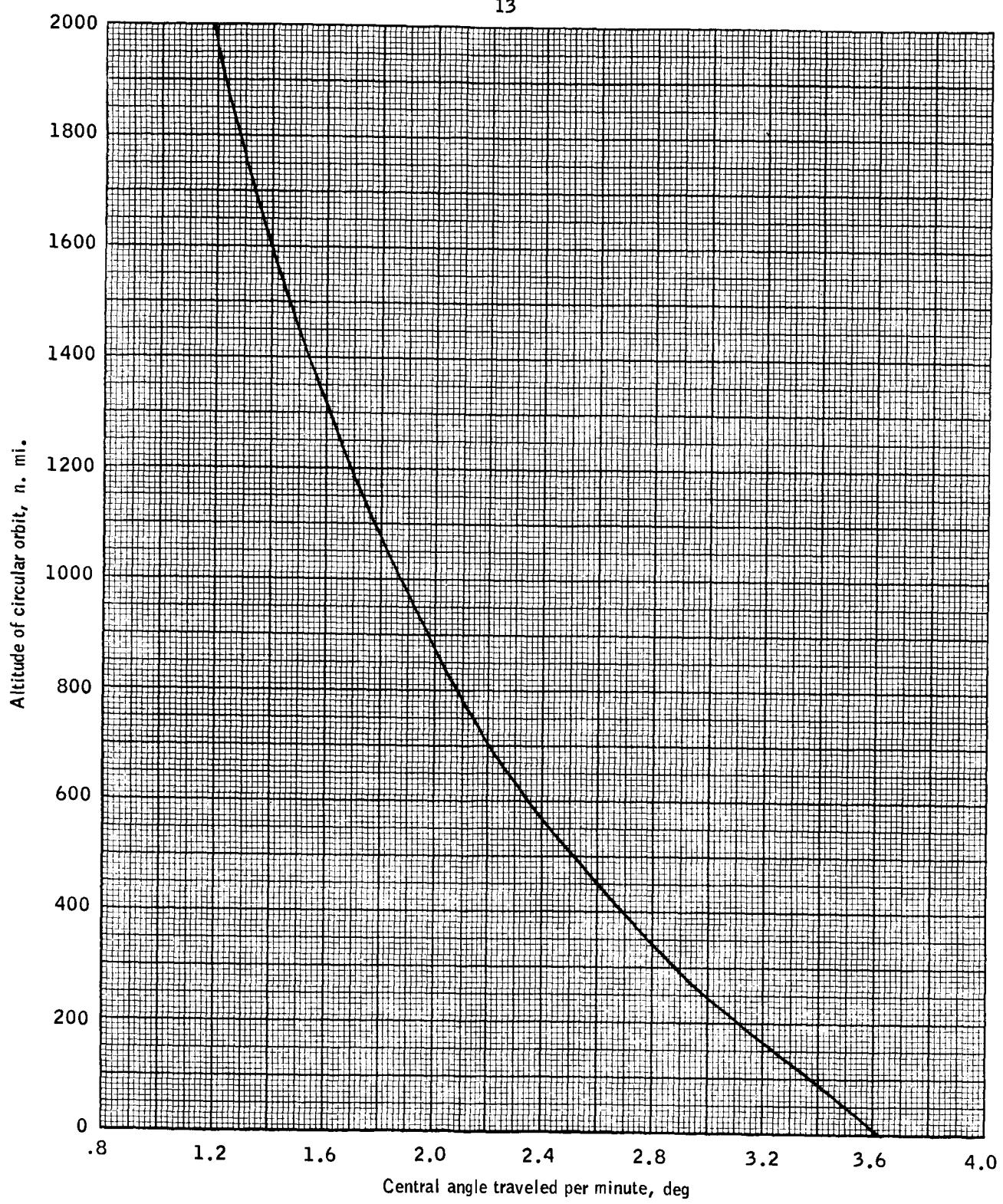


Figure 4.- Altitude of circular Mars orbit versus degrees of central angle traveled per minute.

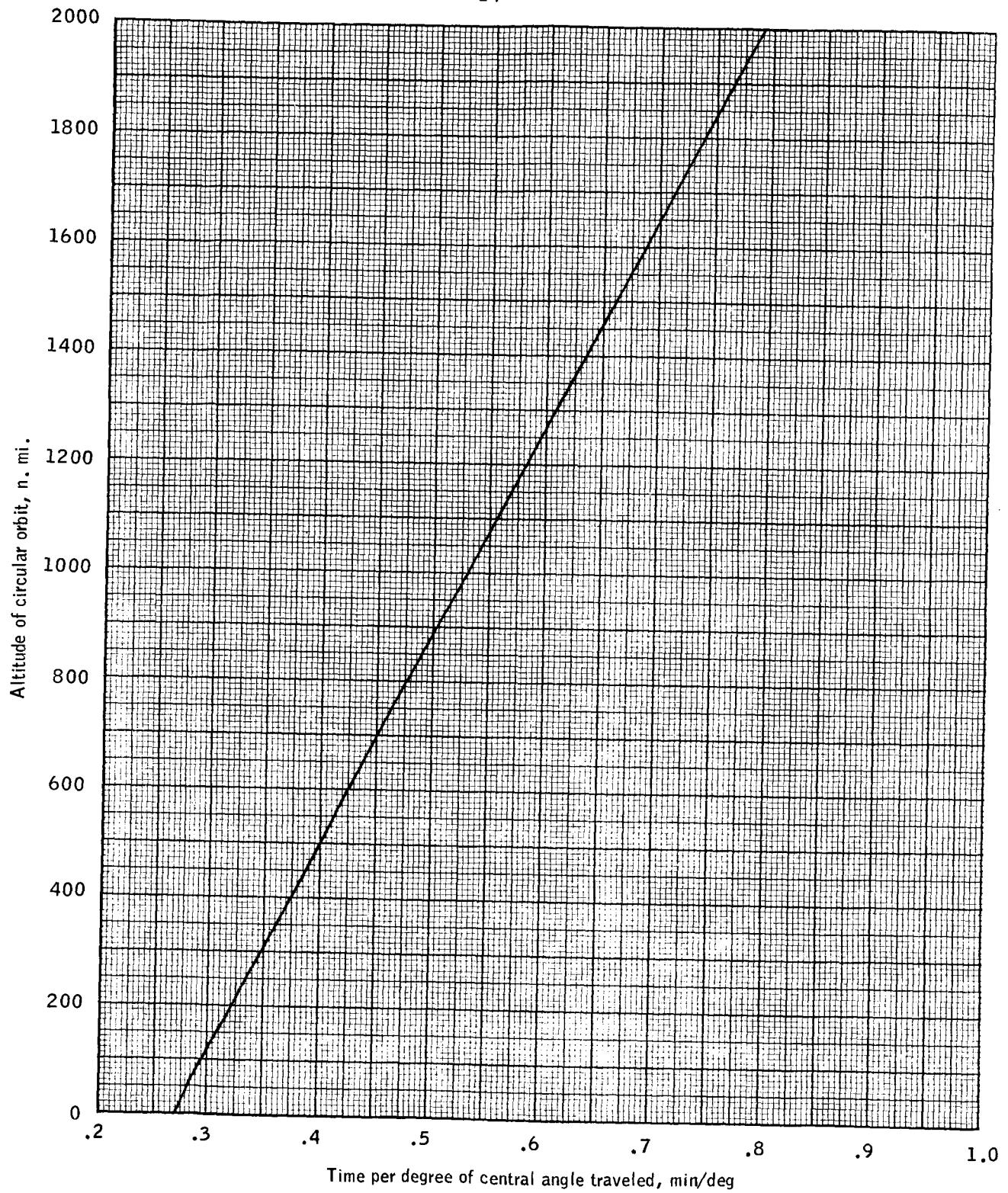


Figure 5.- Altitude of circular Mars orbit versus minutes per degree of central angle travel.

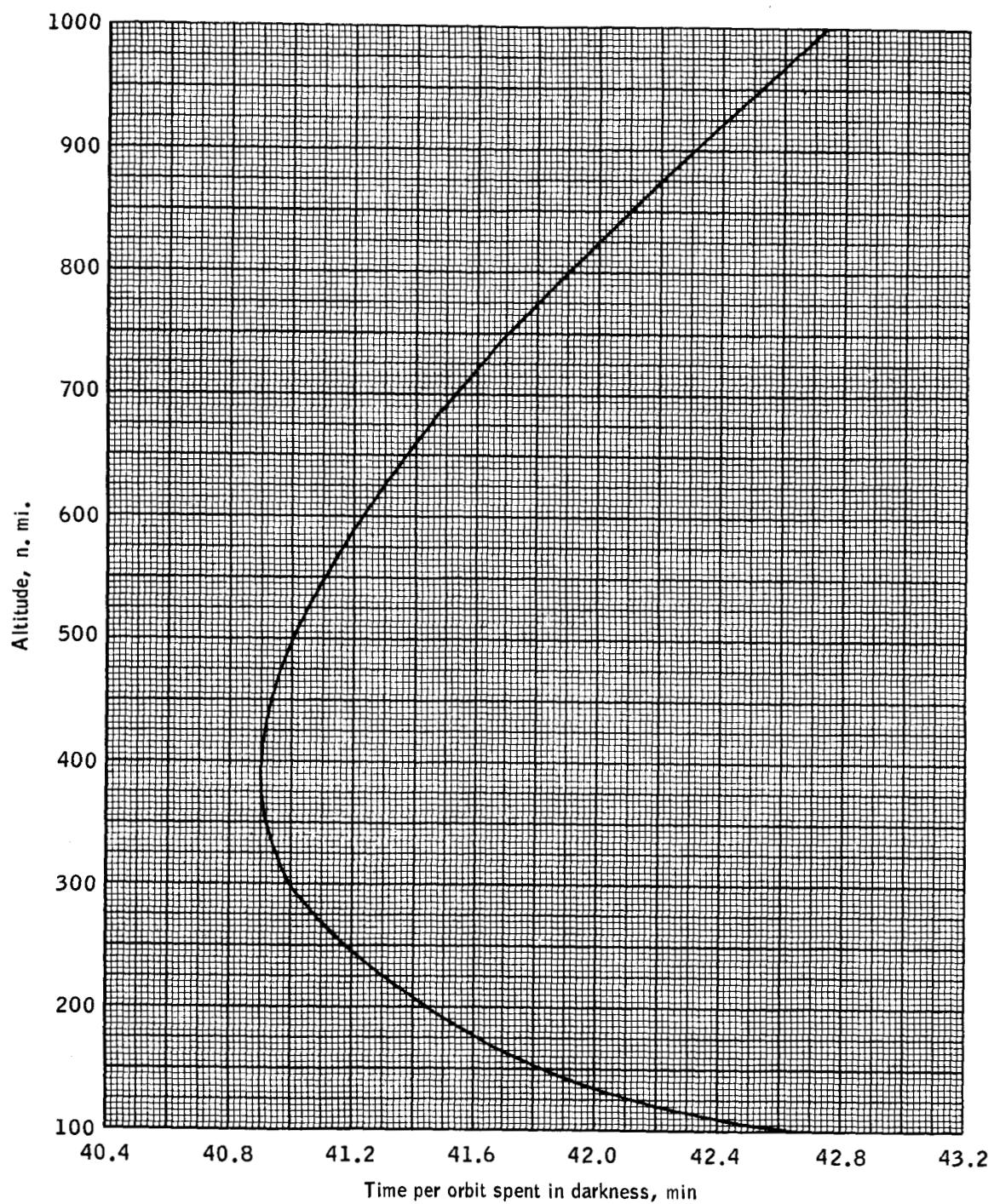


Figure 6.- Altitude of circular Mars orbit versus minutes spent in darkness per orbit for zero inclination to the orbital plane of Mars.

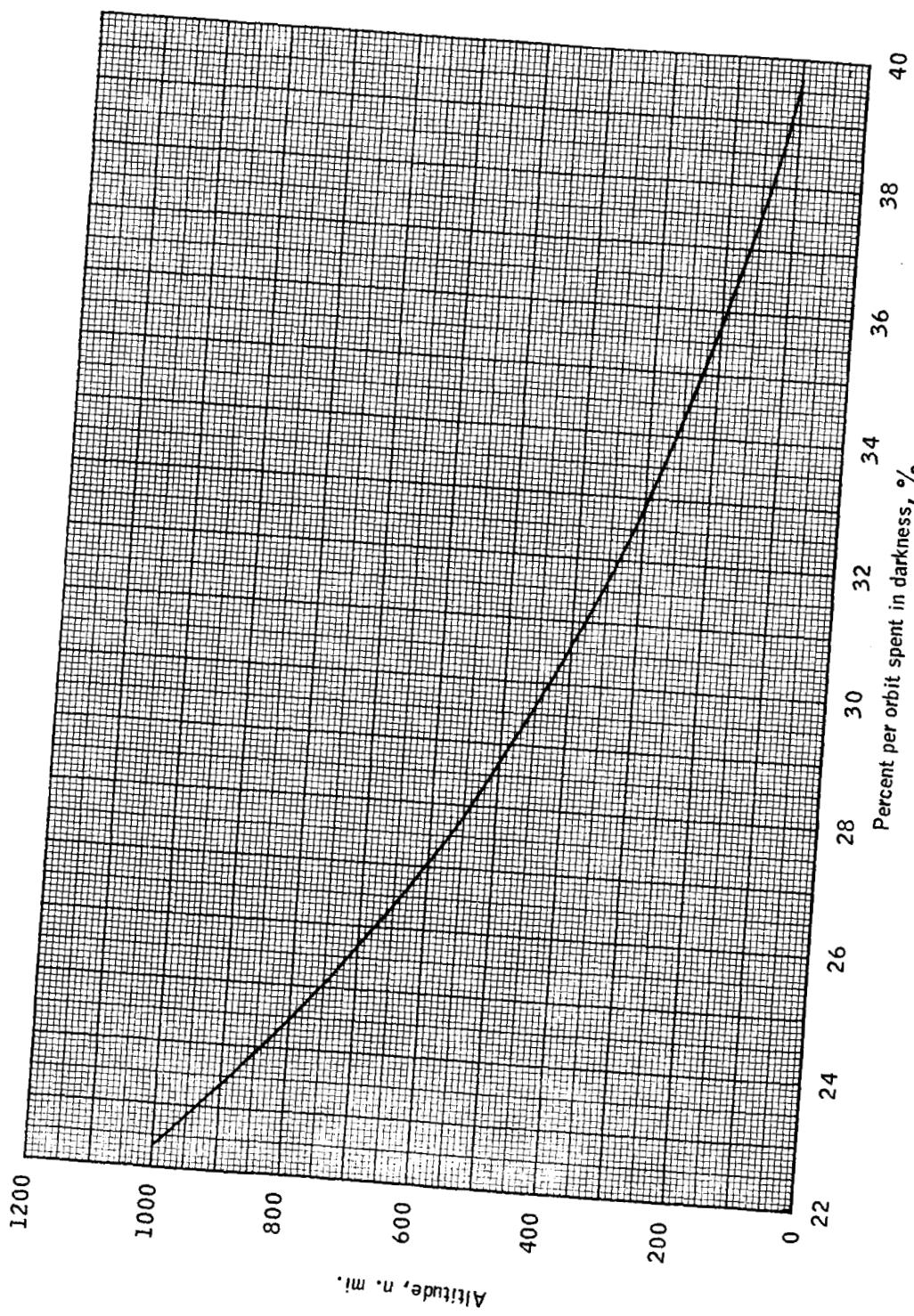


Figure 7.- Altitude of circular Mars orbit versus percentage of one orbit spent in darkness for zero inclination to the orbital plane of Mars.

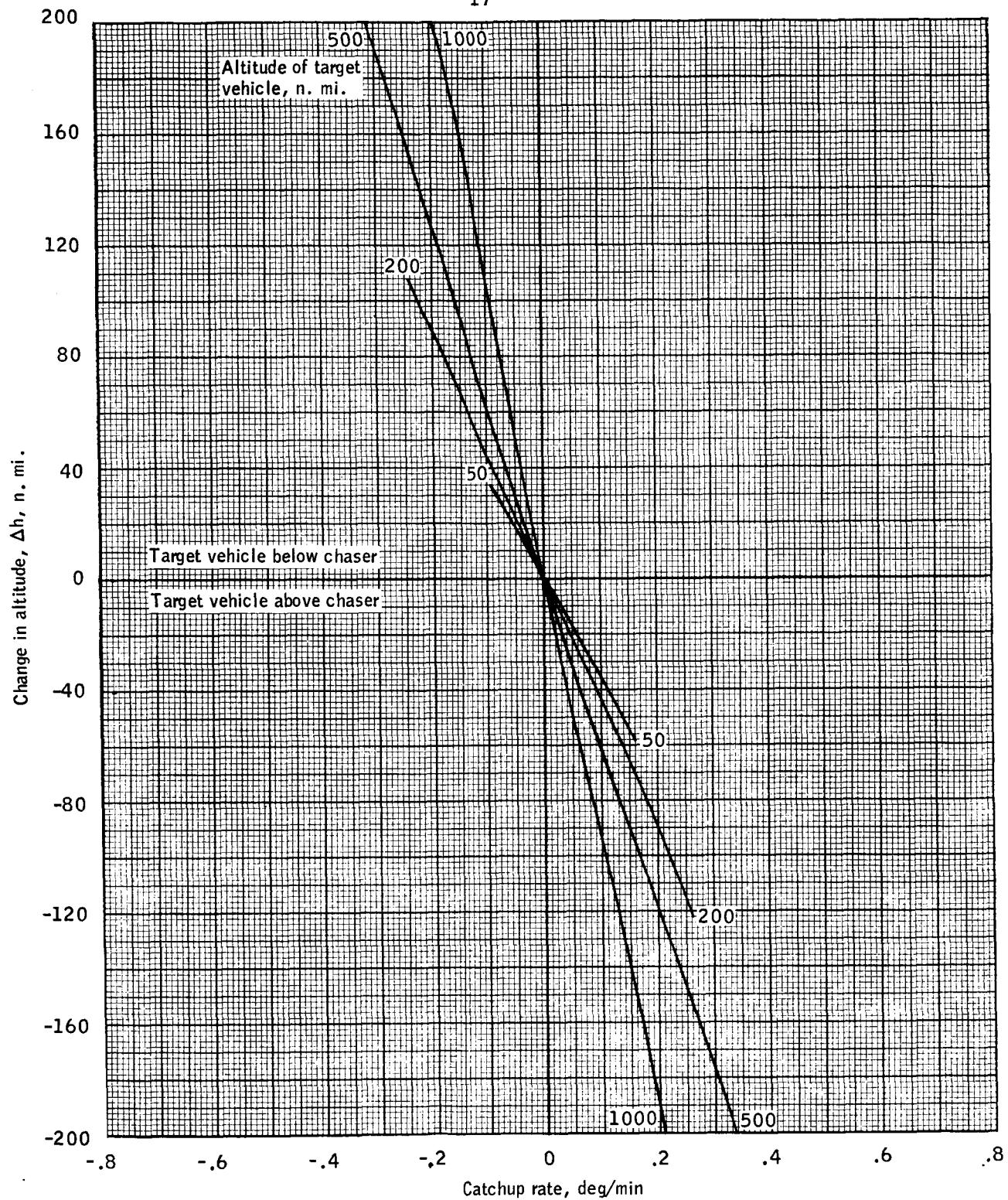


Figure 8.- Catchup rate versus Δh for a target vehicle in a circular orbit around Mars.

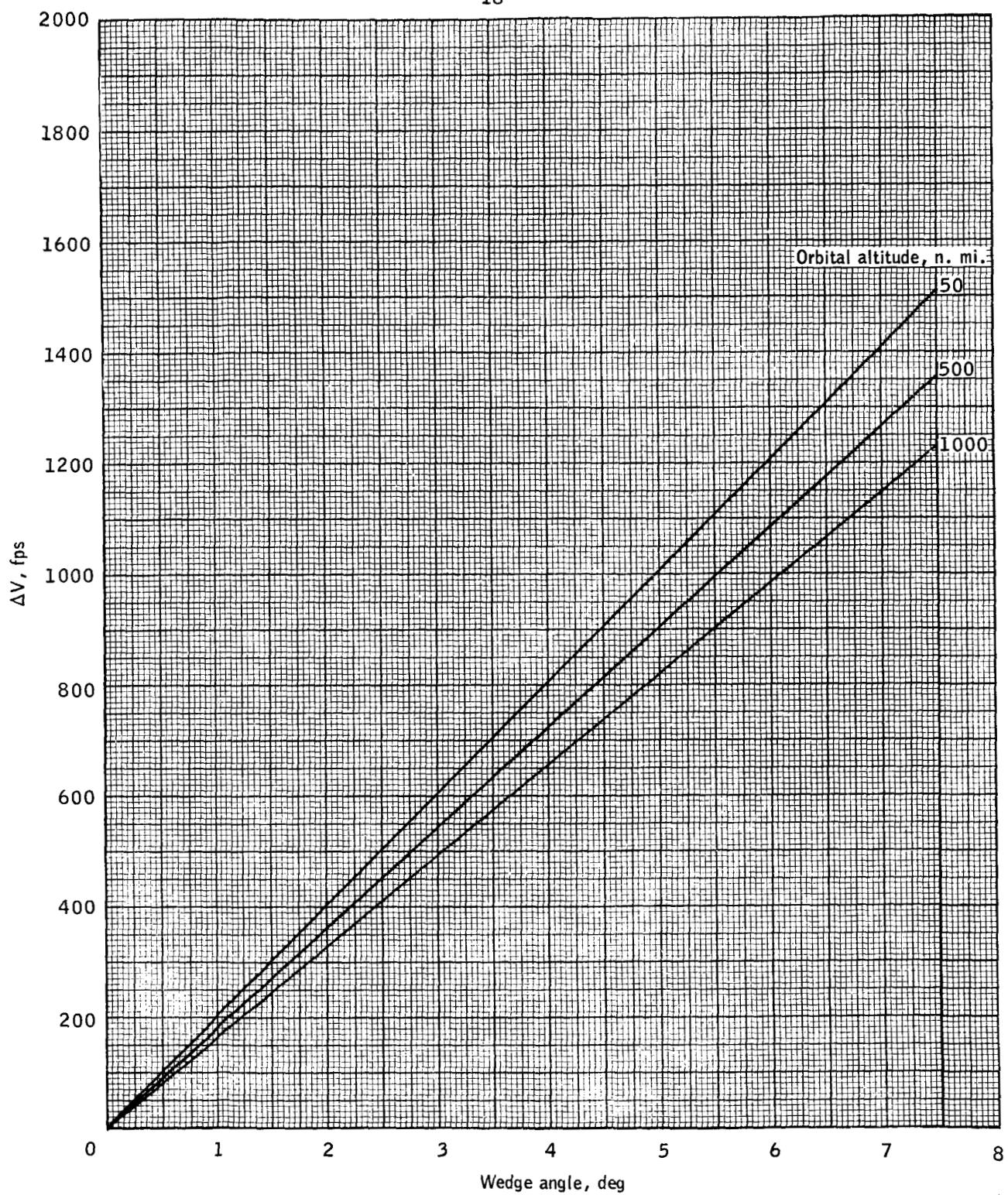


Figure 9.- ΔV requirements for plane changes for 50- to 1000-n. mi. circular orbits around Mars.

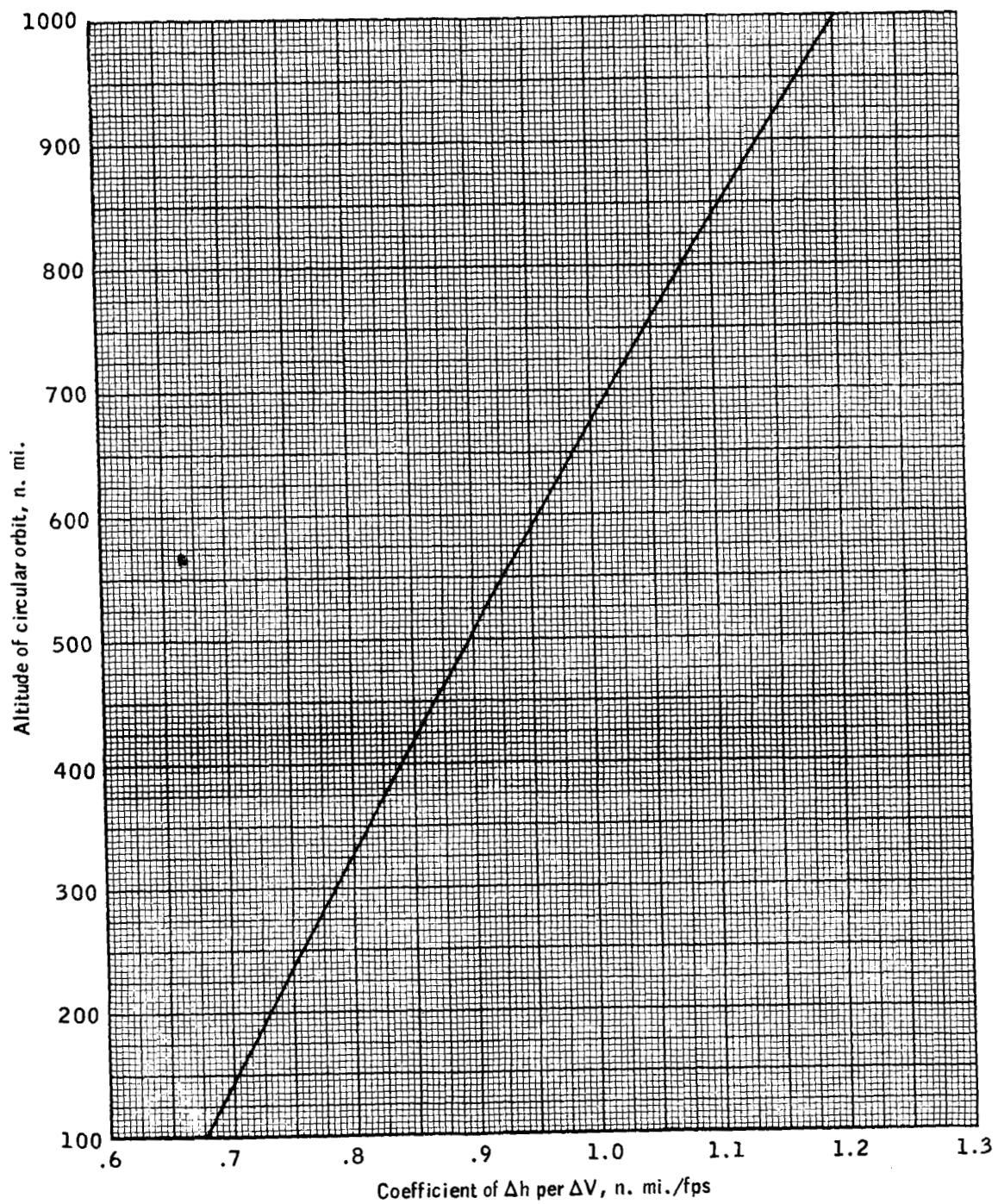


Figure 10.- Ratio of Δh to ΔV for Hohmann transfers versus altitude of circular Mars orbit.

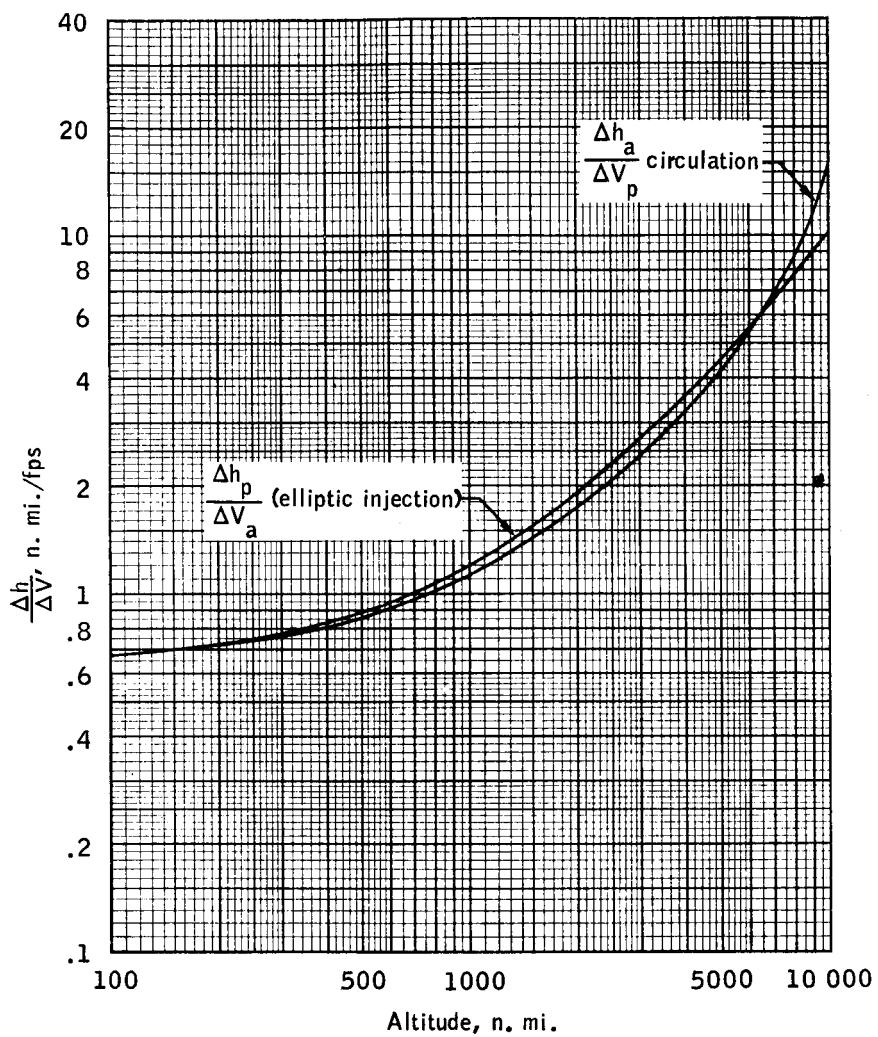


Figure 11.- Ratio of Δh to ΔV for Hohmann transfers versus altitude of elliptic Mars orbit.

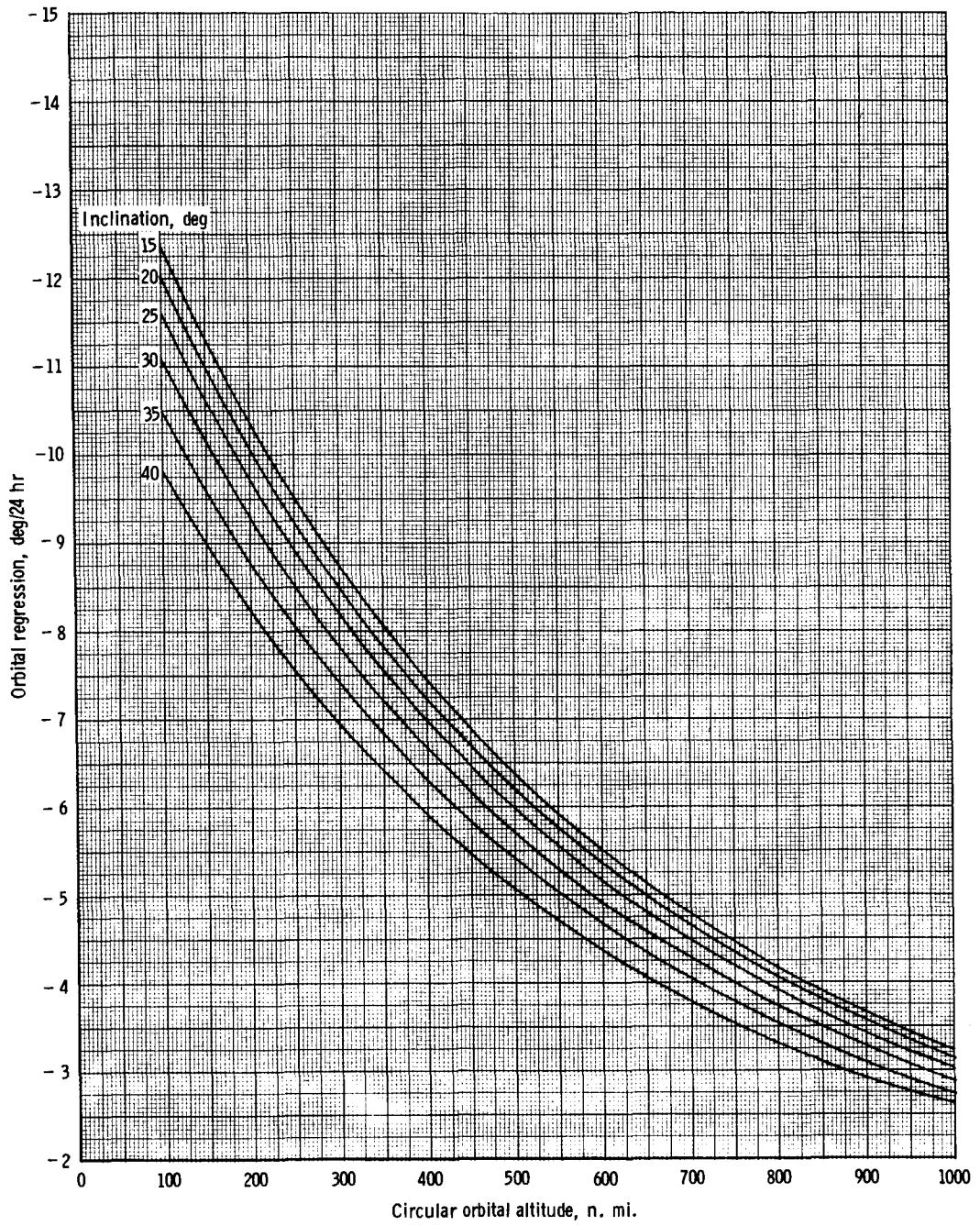


Figure 12.- Orbital regression for various inclinations as a function of circular orbital altitudes.

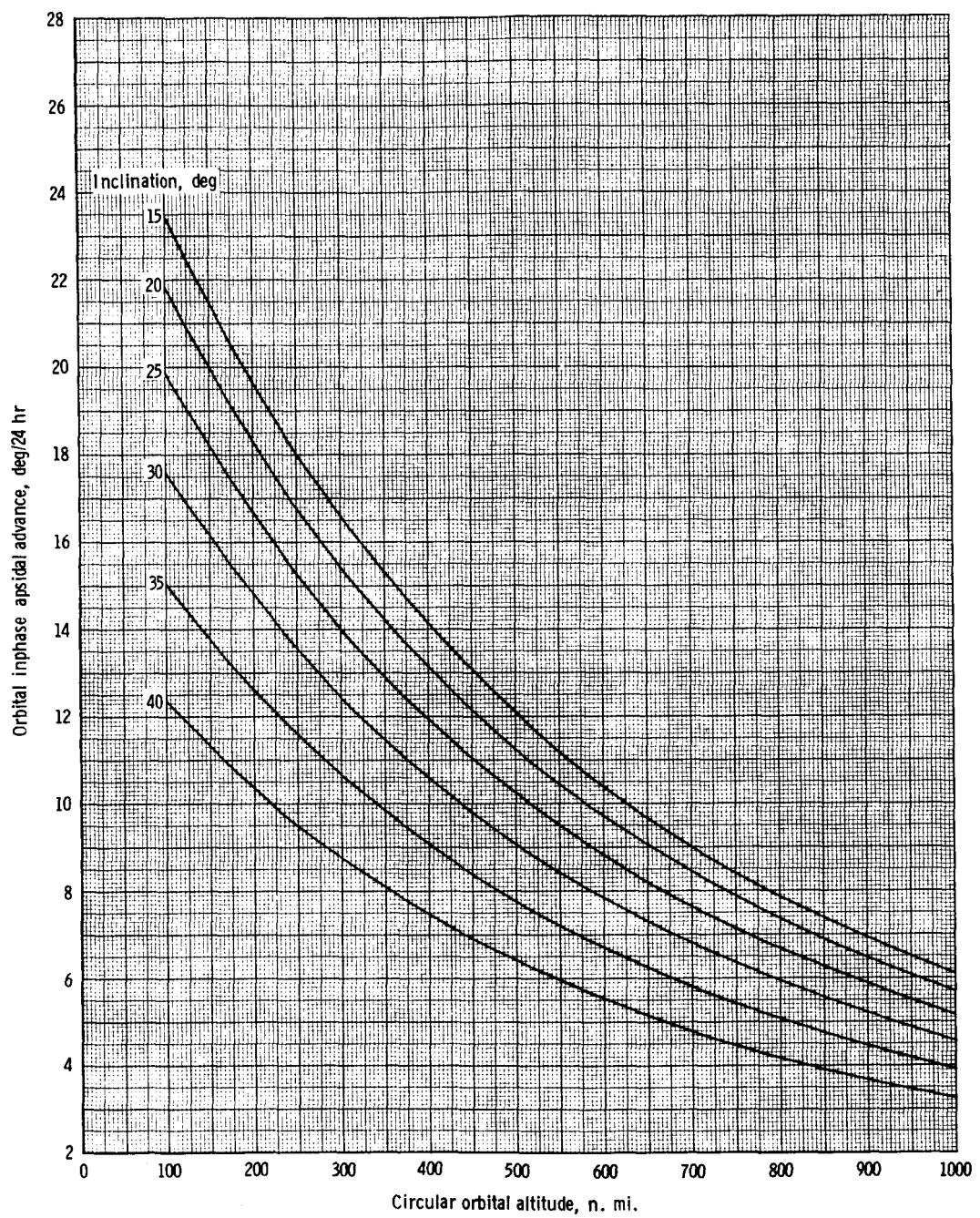


Figure 13.- Orbital inplane apsidal advance for various inclinations as a function of circular orbital altitudes.

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